**Assignment #3 – CCDs and Detectors (42 marks total)**

1. [**5 marks**]
2. Constellation: Ursa Major **[1]**
3. Plate (or focal-plane) scale: (ΔRA”2+ ΔDec”2)1/2 / (Δcol2 + Δrow2)1/2

ΔRA” = 15 (53.13-21.32) × cos(32.8) = 401.1” ΔDec” = 60 (52-49) + 36 = 216”

Δcol = 821.5 – 311.7 = 509.8 px Δrow = 633.2 – 247.9 = 385.3 px

So plate scale = (401.1”2 + 216”2)1/2 / (509.82 + 385.32)1/2 = 455.6/639.0 = 0.713”/px **[2]**

|  |  |
| --- | --- |
| 1. In the figure at right, the *x,y* axes represent the RA, Dec or α,δ coordinate system and the (*x’, y’*) axes represent the columns, rows of the CCD. If *p*1 and *p*2 are the two stars, the distance between the two stars, Δ*s,* is invariant or the same length in both systems. The angle between the two *x* or *y* axes, the rotation of the coordinate system is θ. Now cos(θxy) = Δ*y*/Δ*s* and cos(θx’y’) = Δ*y’*/Δ*s*. This yields θxy = 61.7 degrees and θx’y’ = 32.3 degrees. Thus the CCD axis is rotated 32.3 – 61.7 = -29.4 degrees. (It’s negative because a + rotation is N through E.) **[2]** |  |

1. [**2 marks**] Rewriting and solving the quadratic equation in *t*,

[1 mark for this solution plus 1 mark for the “+” sign (and not +/- since “-“ is unphysical.]

1. [**3 marks; 0.5 mark for {} expression, 0.5 mark for time dependence**] If *T* = *B + D*, then using (1):
2. For very bright objects with *F* >> *T, R*, then σ1 = *F t* /(*Ft)*1/2 = {(*Ft)*1/2} ∝ *t*1/2
3. For objects in which the sky dominates, *T* >> R, *F* <<*T,* then σ1 = *F t* /(*Tt)*1/2 = {*F/√T* (*t)*1/2} ∝ *t*1/2
4. For read-noise dominated objects with *R* >> *(F+T)t* then σ1 = {*F t* /*R*} ∝ *t*
5. [**1 mark**] For a uniformly distributed signal over *N* pixels, σN = σ1 (*N*)1/2
6. [**5 marks**] Now this is really an academic question. Chips of this size normally have multiple readout options (e.g., serial registers and amplifiers in each corner), and so 4096 transfers are not feasible (let alone three-phase readouts which require in this hypothetical question 3 × 4096 transfers). Nevertheless, let’s continue under this assumption. Now the minimum charge transfer efficiency, CTE, required so that X percent of the electrons make it to the serial register, then CTE = (X/100)1/(3 × 4096)
7. [**3 marks**; **1 for equation above and 0.5 for each Minimum CTE.**] The results are expressed in this table (and there must be minimum of 9 significant figures for each):

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Percent e- | 99 | 99.5 | 99.9 | 99.99 |
| Minimum CTE | 0.999 999 182 | 0.999 999 592 | 0.999 999 9186 | 0.999 999 9919 |

1. [**2 marks; 0.5 marks each**] If there are 10,000 e- at the bottom row, then the number of “lost” electrons at the serial register is: (1 – X/100) \* 10,000.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Percent e- | 99 | 99.5 | 99.9 | 99.99 |
| No. e- lost | 100 | 50 | 10 | 1 |

1. [**5 marks; 0.5 mark for each “section”.**] Using equation (1);

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| R=10 e-/px, **F=100 e-/s/px** | **t(s)**  **σ1 = 2** | **t(s)**  **σ1 = 5** | **t(s)**  **σ1 = 10** | **t(s)**  **σ1 = 20** | **t(s)**  **σ1 = 50** |
| T/F = 0.01 | |  | | --- | | 0.22 | | 0.64 | 1.63 | 4.86 | 26.20 |
| T/F = 0.1 | |  | | --- | | 0.22 | | 0.66 | 1.69 | 5.17 | 28.38 |
| T/F = 1 | |  | | --- | | 0.24 | | 0.81 | 2.41 | 8.47 | 50.50 |
| T/F = 10 | |  | | --- | | 0.52 | | 2.84 | 11.09 | 44.09 | 275.09 |
|  |  |  |  |  |  |
| R=10 e-/px, **F=5 e-/s/px** | **t(s)**  **σ1 = 2** | **t(s)**  **σ1 = 5** | **t(s)**  **σ1 = 10** | **t(s)**  **σ1 = 20** | **t(s)**  **σ1 = 50** |
| T/F = 0.01 | 4.42 | 12.84 | 32.51 | 97.25 | 524.08 |
| T/F = 0.1 | 4.46 | 13.12 | 33.83 | 103.46 | 567.62 |
| T/F = 1 | 4.88 | 16.18 | 48.28 | 169.44 | 1009.90 |
| T/F = 10 | 10.35 | 56.76 | 221.80 | 881.81 | 5501.82 |

1. [**21 marks**]
2. Field of view (FOV): [**2 marks**; **1 mark each**]

2 (detectors) × 2048 (pixels/detector) × 0.3 (arcsec/pixel) / 60 (arcsec/arcmin) = 20.48‘

Plate scale: 0.3 (arcsec/px) × 1000 (mm/micron) / 18 micron = 16.67 arcsec/mm

1. Total monochromatic energy flux of a star at the centre of the filter: [**2 marks**; **1 for equation and 1 for three reasonably correct entries**]

*Fν(J)* = *fν(J)* × (π (*D*/2)2 ) × 10-0.4(*m*- *mo)* W/Hz where *D* = 3.6 m, the diameter of the primary mirror of the CFHT, and *mo* is the monochromatic magnitude of the standard which is 0 here. So…

|  |  |
| --- | --- |
| Filter | Monochromatic energy flux |
| *J* | 1.68 × 10-22 dex(-0.4 m*J*) W/Hz |
| *H* | 1.09 × 10-22 dex(-0.4 m*H*) W/Hz |
| *Ks* | 6.85 × 10-23 dex(-0.4 m*Ks*) W/Hz |

where “dex(a)” refers to 10a

1. The total energy flux of a star of magnitude m through each of the files is *Fν* Δν W or *Fν* Δλ *c*/λ2 which yields: [**2 marks**; **1 for equation and 1 for three reasonably correct entries**]

|  |  |
| --- | --- |
| Filter | Total energy flux |
| *J* | 1.01 × 10-8 dex(-0.4 m*J*) W |
| *H* | 3.75 × 10-9 dex(-0.4 m*H*) W |
| *Ks* | 1.70 × 10-9 dex(-0.4 m*Ks*) W |

1. The photon flux through a filter is the energy flux divided by (*h ν*), the energy of a photon at the centre of a filter. [**2 marks**; **1 for equation and 1 for three reasonably correct entries**]

|  |  |
| --- | --- |
| Filter | (Raw) Photon flux |
| *J* | 6.12 × 1010 dex(-0.4 m*J*) photon/s |
| *H* | 2.87 × 1010 dex(-0.4 m*H*) photon/s |
| *Ks* | 1.88 × 1010 dex(-0.4 m*Ks*) photon/s |

1. The number of photon/s recorded by the HgCdTe array from a star of apparent magnitude *m* is the product of the photon flux and the total efficiency of the system in the waveband, so: [**2 marks**; **1 for equation and 1 for three reasonably correct entries**]

|  |  |  |
| --- | --- | --- |
| Filter | Efficiency | Recorded photon flux (star) |
| *J* | 0.41 | 2.51 × 1010 dex(-0.4 m*J*) photon/s |
| *H* | 0.52 | 1.49 × 1010 dex(-0.4 m*H*) photon/s |
| *Ks* | 0.53 | 1.00 × 1010 dex(-0.4 m*Ks*) photon/s |

1. The number of photon/s recorded by the array from the sky background is the product of the photon flux and the total efficiency of the system in the waveband, so: [**2 marks**; **1 for equation and 1 for three reasonably correct entries**]

|  |  |  |
| --- | --- | --- |
| Filter | Sky Brightness | Recorded photon flux (sky) |
| *J* | 16.7 | 472 photon/s |
| *H* | 15.4 | 931 photon/s |
| *Ks* | 15.4 | 620 photon/s |

1. The maximum exposure time is the time required to reach the 5% linearity level of 20,000 ADU which is 20,000 × gain = 20,000 ADU × 3.8 e-/ADU = 76,000 e- per pixel from all sources, i.e., source + sky background (since the dark current is negligible). All the signals, background + source fall on (0.6/0.3)2 = 4 pixels uniformly by assumption. The following table contains these results: [**5 marks**; **2 marks for equations/logic and 1 for each reasonably correct t(s)/column**]

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | *F(J)* | *t* | σ | σ | *F(H)* | *T* | σ | σ | *F(K*s*)* | *t* | σ | σ |
| mag | phot/s/px | s | total | 1 hr | phot/s/px | S | total | 1 hr | phot/s/px | s | total | 1 hr |
| 15 | 15304.6 | 11.3 | 798.9 | 14282 | 7186.9 | 16.3 | 9074 | 14.9 | 4696.5 | 24.4 | 604.4 | 7335 |
| 16 | 6092.9 | 25.6 | 722.5 | 8569 | 2861.2 | 31.4 | 5019 | 10.7 | 1869.7 | 47.2 | 464.4 | 4057 |
| 17 | 2425.6 | 51.8 | 582.5 | 4855 | 1139.1 | 49.9 | 2517 | 8.5 | 744.3 | 74.9 | 293.6 | 2035 |
| 18 | 965.7 | 87.6 | 391.9 | 2512 | 453.5 | 65.1 | 1145 | 7.4 | 296.3 | 97.8 | 152.6 | 925 |
| 19 | 384.4 | 120.7 | 215.1 | 1174 | 180.5 | 74.2 | 486 | 7.0 | 118.0 | 111.4 | 69.2 | 393 |
| 20 | 153.0 | 142.1 | 100.8 | 507 | 71.9 | 78.5 | 199 | 6.8 | 47.0 | 117.9 | 29.1 | 161 |
| 21 | 60.9 | 152.9 | 43.2 | 209 | 28.6 | 80.4 | 80 | 6.7 | 18.7 | 120.7 | 11.9 | 65 |
| 22 | 24.3 | 157.7 | 17.7 | 85 | 11.4 | 81.1 | 432 | 6.7 | 7.4 | 121.8 | 4.8 | 26 |
| 23 | 9.7 | 159.7 | 7.1 | 34 | 4.5 | 81.4 | 13 | 6.6 | 3.0 | 122.3 | 1.9 | 10 |

1. The total S/N ratio, σtotal = σ1 × [(0.6/0.3)2]1/2 = 2 σ1 and is recorded in the table above [**2 marks; 0.5 mark for equation and 0.5 marks for each reasonably correct column**]
2. The S/N ratio of the entire star for a sequence of exposures whose *total* exposure time is one hour or 3,600 s is σ1hr = (3600/*t*)1/2 σtotal (though one should really only take the integer of this quotient) and is recorded in the table above. [**2 marks; 0.5 mark for equation and 0.5 mark for each reasonably correct column**] Note that high sign-to-noise ratios are always smaller than this since no array is a perfect device and a real PSF is not square!